Testing series (which tests could be used?)

① $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$ — $\frac{1}{n+3^n} < \frac{1}{3^n}$ Conv. by C.T. (Abs.Co.)

② $\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$ — $\sqrt[n]{|a_n|} = \frac{2n+1}{n^2} \to 0 < 1$ abs. Con. by Root tes.

③ $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+2}$ — Con. by A.S.T. $\left(f'(x) = \frac{2-x^2}{(x^2+2)^2}\right)$ Condit. Conv. since $\sum |a_n|$ is div. by L.C.T. $(b_n = \frac{1}{n})$

④ $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$ — $\sum |a_n|$ is div. by Test for div. $\Rightarrow \lim_{n\to\infty} (-1)^n \frac{n}{n+2} = \pm 1 \neq 0 \Rightarrow$ div. by T. for div.

⑤ $\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$ — $\left|\frac{a_{n+1}}{a_n}\right| = \frac{2(n+1)^2}{5n^2} \to \frac{2}{5} < 1 \Rightarrow$ abs. Conv. by Ratio test.

⑥ $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ — div. by L.C.T. $b_n = \frac{1}{n}$

⑦ $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ — div. by Integral test

⑧ $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$ — $\left|\frac{a_{n+1}}{a_n}\right| = \frac{2(k+1)}{k+3} \to 2 > 1$ div. by Ratio test.

⑨ $\sum_{k=1}^{\infty} k^2 e^{-k}$ — $\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{e}\left(\frac{k+1}{k}\right)^2 \to \frac{1}{e} < 1$ abs. Conv. by ratio test

⑩ $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ — $\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{e^{3n^2+3n+1}} \left(\frac{n+1}{n}\right)^2 \to 0.1 = 0 < 1$ abs. Conv. by ratio test

⑪ $\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n}\right)$ — sum of 2 conv. series

⑫ $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k^2+1}}$ — Conv. by C.T. $\frac{1}{k\sqrt{k^2+1}} < \frac{1}{k\sqrt{k^2}} = \frac{1}{k^2}$

⑬ $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ — $\left|\frac{a_{n+1}}{a_n}\right| = \frac{3(n+1)^2}{(n+1)n^2} \to 0 < 1$ abs. Conv. (ratio test)

⑭ $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$ — $\left|\frac{\sin 2n}{1+2^n}\right| \leq \frac{1}{1+2^n} < \frac{1}{2^n}$ abs. Conv. by C.T.

⑮ $\sum_{k=1}^{\infty} \frac{2^{k-1} 3^{k+1}}{k^k} = \sum_{k=1}^{\infty} \frac{3}{2}\left(\frac{2\cdot 3}{k}\right)^k$, $\sqrt[k]{|a_k|} = \frac{6}{k} \to 0 < 1$ abs. Conv. (root test)

⑯ $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ — div. by L.C.T. $b_n = \frac{1}{n}$

⑰ $\sum_{n=0}^{\infty} \frac{n!}{2\cdot 5\cdot 8 \cdots (3n+2)}$ — $\left|\frac{a_{n+1}}{a_n}\right| = \frac{n+1}{3n+2} \to \frac{1}{3} < 1$ abs. Conv. (Ratio test)

⑱ $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$ — Conditionally convergent. by A.S.T. $\lim_{n\to\infty} b_n = 0$, decreasing but $\sum_{n=1}^{\infty} |a_n|$ is div. by C.T.

(19) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

$f'(x) = \frac{2-\ln x}{2x^{3/2}} < 0$ for $x > e^2$ conv. by A.S.T.

but $\frac{\ln n}{\sqrt{n}} > \frac{1}{\sqrt{n}}$ div. by C.T. (Conditionally) Conv.

(20) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

L.C.T. $b_n = \frac{1}{n^2}$ $\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$

(21) $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n^2}\right)$

$\lim_{n\to\infty} \cos\frac{1}{n^2} = \cos 0 = 1 \Rightarrow \lim_{n\to\infty} a_n = \pm 1$ div by T. for d.

(22) $\sum_{k=1}^{\infty} \frac{1}{2+\sin k}$

$-1 \le \sin k \le 1 \Rightarrow 1 \le 2+\sin k \le 3 \Rightarrow \frac{1}{3} \le \frac{1}{2+\sin k} \le 1$

$\lim_{n\to\infty} a_n \neq 0$ div by T. for div.

(23) $\sum_{n=1}^{\infty} \tan\frac{1}{n}$

$\lim_{n\to\infty} \tan\frac{1}{n} = 0$, Div by L.C.T. $b_n = \frac{1}{n}$, $\lim_{x\to\infty} \frac{\tan 1/x}{1/x}$

$= \lim_{x\to\infty} \frac{(\sec^2\frac{1}{x})(-\frac{1}{x^2})}{(-1/x^2)} = 1 > 0$.

(24) $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

$\lim_{n\to\infty} n \sin\frac{1}{n}$ $\infty \cdot 0$ $\Rightarrow \lim_{x\to\infty} \frac{\sin 1/x}{1/x} = \lim_{x\to\infty} \cos\frac{1}{x} = 1 \neq 0$ Div. by Test for div.

(25) $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{n+1}{e^{2n+1}}$ by H $\to 0 < 1$ abs. Conv.

(26) $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{5} \cdot \frac{n^2+2n+2}{n^2+1} \to \frac{1}{5} < 1$ abs. Conv.

(27) $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$

$\frac{k\ln k}{(k+1)^3} < \frac{k\ln k}{k^3} = \frac{\ln k}{k^2}$ (Conv. by C.T.) $\to$ or Integral test $\boxed{\ln k < \sqrt{k}}$

(28) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

$\frac{1}{n} < 1 \Rightarrow e^{\frac{1}{n}} < e \Rightarrow \frac{e^{\frac{1}{n}}}{n^2} < \frac{1}{n^2}$ Conv. by C.T.

(29) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n}$

$\sum |a_n| = \sum \frac{1}{\cosh n} = \sum \frac{2}{e^n+e^{-n}}$, $\frac{2}{e^n+e^{-n}} < \frac{2}{e^n}$ $\left(\cosh x = \frac{e^x+e^{-x}}{2}\right)$

Conv. by C.T. $\Rightarrow$ abs. Conv.

(30) $\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$

Conditionally Conv. $\sum |a_j|$ is div by L.C.T. $b_n = \frac{1}{\sqrt{j}}$

while $\sum a_n$ is Conv. by A.S.T. $f'(x) = \frac{5-x}{2\sqrt{x}(x+5)^2}$

(31) $\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$

$\lim_{k\to\infty} \frac{5^k}{3^k+4^k} = \lim_{k\to\infty} \frac{5^k}{4^k} = \lim_{k\to\infty} \left(\frac{5}{4}\right)^k = \infty$ since $\frac{5}{4} > 1$

$\Rightarrow$ div by test for div.

(32) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

$\sqrt[n]{|a_n|} = \frac{n!}{n^4} \to \infty$ div.

(33) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

$\sqrt[n]{|a_n|} = \left(\frac{n}{n+1}\right)^n = \left[\left(\frac{n+1}{n}\right)^n\right]^{-1} \to e^{-1} < 1$ abs. Conv.

$\boxed{\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = e}$

(34) $\sum_{n=1}^{\infty} \frac{1}{n+n\cos^2 n}$

$\cos^2 n < 1 \Rightarrow n\cos^2 n < n \Rightarrow n+n\cos^2 n < 2n$

$\frac{1}{n+n\cos^2 n} > \frac{1}{2n}$ div. by C.T.

(35) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)$

$b_n = \frac{1}{n}$, $\lim_{n\to\infty} \frac{a_n}{b_n} = \ln 2 > 0$ div. by L.C.T.

$\lim_{x\to\infty} \frac{2^{\frac{1}{x}} - 1}{1/x} \overset{H}{=} \lim_{x\to\infty} \frac{2^{\frac{1}{x}} \ln 2 \left(\frac{-1}{x^2}\right)}{-1/x^2} = \ln 2$